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MAM1020S **Test practice** **August 2017**

Very similar question will appear on the test next week. I will not ask anything harder. Try them all out, use helpnotes, slides and tutorial solutions. If you still do not understand or manage answering, write down the question number and we can do it together on Friday. If you want to double check if your answer is correct because still with the helpnotes, slides or tutorial solutions you are not sure ASK ME before the test. Remember you are smart enough to do anything, it sometimes just take a bit of time and courage. NO SOLUTION WILL BE POSTED.

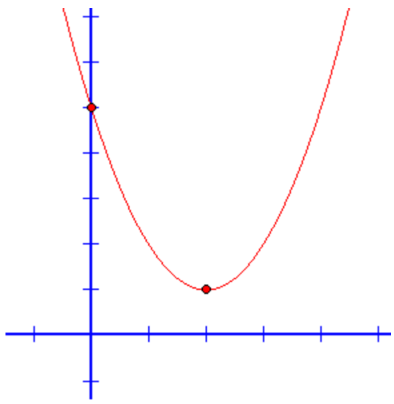
1. Is the set of points (x, y) in the cartesian plane such that $x^2 + y^2 = 3$ the graph of a function in x?

2. Is the set of points (x, y) in the cartesian plane such that $y = x^2 + 3x + 6$ the graph of a function in x?

   BE CAREFUL, DO NOT MAKE THE CONFUSION : *Being a function means that you have only one y for each x. That is* **totally different** *from being one-to-one which is having at most one x for each y. (Function is about uniqueness of y, one-to-one is about x.:-))*

3. Is
$$\begin{array}{rccc} f: & (0, +\infty) & \to & \mathbb{R} \\ & x & \to & x + \frac{5}{x} \end{array}$$
   a polynomial function? a rational function?

4. Find $f(x)$ quadratic polynomial whose graph is



5. Let
$$\begin{array}{rccc} f: & \mathbb{R} & \to & \mathbb{R} \\ & x & \to & 6x + 5 \end{array},$$

   (a) What is the domain and the codomain of f?
   (b) Compute the image of 2 by f.
   (c) Compute the preimage of 3 by f.
   (d) Give the definition of $Graph(f)$.
   (e) Is $(1, 2)$ a point of the Graph of f ?
   (f) Prove that f is onto.
   (g) Prove that f is one-to-one.

(h) Is f bijective?

(i) Compute the inverse of f.

(j) Verify that the inverse computed in the previous question is indeed an inverse using the identity seen in class for inverses with composition.

(k) Prove that f is increasing.

(l) Prove that f is not odd.

(m) When $0 \leq x \leq 1$, what is the range of values of $f(x)$ (meaning I am asking you to find a, b real numbers such that $a \leq f(x) \leq b$ when $0 \leq x \leq 1$). Deduce $f([0,1])$.

(n) What is the range of x in the domain, when we have $0 \leq f(x) \leq 1$ (meaning I am asking you to find a, b reals numbers such that if $0 \leq f(x) \leq 1$ then $a \leq x \leq b$.) Deduce $f^{-1}([0,1])$.

6. Let

$$f: \begin{array}{ccc} \mathbb{R} & \rightarrow & \mathbb{R} \\ x & \rightarrow & x^2+3 \end{array},$$

(a) What is the domain and the codomain of f?

(b) Compute the image of 2 by f.

(c) Compute the preimage of 3 by f.

(d) Prove that f is not onto.

(e) Prove that f is not one-to-one.

(f) Is f bijective?

(g) Prove that f is not increasing.

(h) Prove that f is even.

7. Let

$$f: \begin{array}{ccc} \mathbb{R} & \rightarrow & \mathbb{R} \\ x & \rightarrow & x^2+3 \end{array},$$

(a) What is the domain and the codomain of f?

(b) Compute the image of 2 by f.

(c) Compute the preimage of 3 by f.

(d) Prove that f is not onto.

(e) Prove that f is not one-to-one.

(f) Is f bijective?

(g) Prove that f is not increasing.

(h) Prove that f is even.

8. Let

$$f: \begin{array}{ccc} \mathbb{R} & \rightarrow & \mathbb{R} \\ x & \rightarrow & \left\{ \begin{array}{cc} 2x & x \leq 0 \\ 2-2x & x > 0 \end{array} \right. \end{array},$$

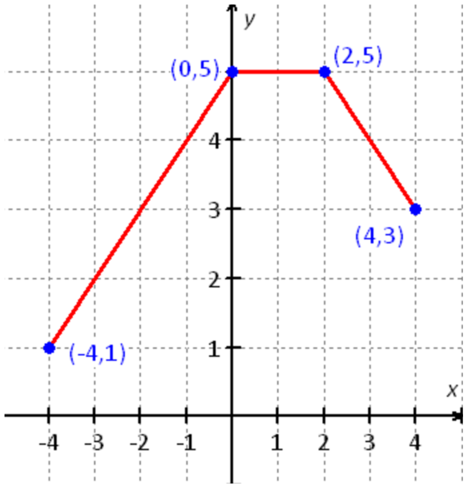
(a) Compute the image of 0, 1 by f.

(b) Compute the preimages of 0 by f.

(c) Prove that this function is not one to 0.

(d) Sketch a graph for this function.

9. Say we have the following graph of f.



   (a) What is the domain of definition of f?
   (b) Graphically, what is the image of 4?
   (c) Graphically, what are the preimages of 3?
   (d) Graphically, find the range of f.
   (e) Graphically, for which values of x do we have $f(x) \geq 0$.
   (f) Graphically, prove that f is not one to one.
   (g) Find the equation for f over $[-4, 0]$, over $[0, 2]$ and over $[2, 4]$.
   (h) Write f as a piece wise function.
   (i) Graphically, find $f([0, 1])$.
   (j) Graphically, find $f^{-1}([0, 1])$.

10. Find the focus of the parabola whose equation is $y = -2x^2$;

11. Write the standard form of the equation of the parabola with vertex at the origin and focus at $(2, 0)$;

12. Sketch the ellipse given by $4x^2 + y^2 = 36$.

13. Find the standard form of the equation of the hyperbola with foci at $(-3, 0)$ and $(3, 0)$ and vertices at $(-2, 0)$ and $(2, 0)$.

14. Sketch the ellipse given by $4x^2 + y^2 = 36$.

15. Sketch the hyperbola whose equation is $4x^2 - y^2 = 16$.

16. Find the vertex and focus of the parabola given by $x^2 - 2x + 4y - 3 = 0$.

17. Sketch the ellipse given by $x^2 + 4y^2 + 6x - 8y + 9 = 0$.

18. Sketch the hyperbola given by

$$y^2 - 4x^2 + 4y + 24x - 41 = 0$$

19. Write the standard form of the equation of the ellipse whose vertices are $(2, -2)$ and $(2, 4)$. The length of the minor axis of the ellipse is 4.

20. $lim_{x\to 1}(2x^3+5)$;

21. $lim_{x\to 2}\frac{x^3-6}{2x-4}$;

22. $lim_{x\to 1}\frac{x^2-1}{x-1}$;

23. $lim_{x\to 1}g(x)$ where

$$g(x) = \begin{cases} x+1, & if\ x \neq 1 \\ \pi, & if\ x = 1 \end{cases}$$

24. $lim_{h\to 0}\frac{(3+h)^2-9}{h}$

25. $lim_{t\to 0}\frac{\sqrt{t^2+9}-3}{t^2}$

26. $lim_{x\to 0}|x|$;

27. $lim_{x\to 0}\frac{|x|}{x}$;

28. Let $[[x]]$ be the largest integer that is less or equal to $x \in \mathbb{R}$; Prove that $lim_{x\to 3}[[x]]$ does not exist.

29. Show that $lim_{x\to 0}x^2 sin(1/x)$.

30. What is wrong with the following equation

$$\frac{x^2+2x+1}{x+1} = (x+1)?$$

Why $lim_{x\to -1}\frac{x^2+2x+1}{x+1} = lim_{x\to -1}(x+1)$ is still true? Compute this limit.

31. Compute $lim_{x\to 0}\sqrt{x+x^2}cos(\pi/x)$.

32. If $4x-9 \leq f(x) \leq x^2-4x+7$ for $x \geq 0$, compute $lim_{x\to 4}f(x)$.

33. Let $f : \mathbb{R} \to \mathbb{R}$ such that

$$f(x) = \begin{cases} x & x < 0 \\ x+1 & x \geq 0 \end{cases}$$

Compute $lim_{x\to 0-}f(x)$, $lim_{x\to 0+}f(x)$ and $f(0)$.